

$$b) \quad m_1 : a = r_m \wedge m_2 : a = r_m \wedge n_1 : a = r_n \wedge n_2 : a = r_n \\ \Rightarrow (m_1 + n_1) : a = r_{m+n} \wedge (m_2 + n_2) : a = r_{m+n}$$

für $a = 4$

$$m_1 : 4 = r_m \wedge m_2 : 4 = r_m \wedge n_1 : 4 = r_n \wedge n_2 : 4 = r_n \\ \Rightarrow (m_1 + 4) : a = r_{m+n} \wedge (m_2 + n_2) : 4 = r_{m+n}$$

$$V_1 \quad m_1 : 4 = r_m \quad \exists p \in \mathbb{N} : 4p = m_1$$

$$V_2 \quad m_2 : 4 = r_m \quad \exists q \in \mathbb{N} : 4q = m_2$$

$$V_3 \quad n_1 : 4 = r_n \quad \exists r \in \mathbb{N} : 4r = n_1$$

$$V_4 \quad n_2 : 4 = r_n \quad \exists v \in \mathbb{N} : 4v = n_2$$

$$B_1 \quad (m_1 + n_1) : 4 = r_{m+n} \quad \exists k \in \mathbb{N} : 4k = m_1 + n_1$$

$$B_2 \quad (m_2 + n_2) : 4 = r_{m+n} \quad \exists l \in \mathbb{N} : 4l = m_2 + n_2$$

Bew:

$$m_1 + n_1 = 4p + 4r = 4 \underbrace{(p+r)}_{\in \mathbb{N}} = 4k$$

$$m_2 + n_2 = 4q + 4v = 4 \underbrace{(q+v)}_{\in \mathbb{N}} = 4l$$

b) für a bel.

$$V_1 \quad m_1 : a = r_m \quad \exists p \in \mathbb{N} : ap = m_1$$

$$V_2 \quad m_2 : a = r_m \quad \exists q \in \mathbb{N} : aq = m_2$$

$$V_3 \quad n_1 : a = r_n \quad \exists r \in \mathbb{N} : ar = n_1$$

$$V_4 \quad n_2 : a = r_n \quad \exists v \in \mathbb{N} : av = n_2$$

$$B_1 \quad (m_1 + n_1) : a = r_{m+n} \quad \exists k \in \mathbb{N} : ak = m_1 + n_1$$

$$B_2 \quad (m_2 + n_2) : a = r_{m+n} \quad \exists l \in \mathbb{N} : al = m_2 + n_2$$

Bew:

$$m_1 + n_1 = ap + ar = a \underbrace{(p+r)}_{\in \mathbb{N}} = ak$$

$$m_2 + n_2 = aq + av = a \underbrace{(q+v)}_{\in \mathbb{N}} = al$$